ALGEBRAIC NOTATION.

1. Algebra is so much like arithmetic that all that you know about addition, subtraction, multiplication, and division, the signs that you have been using and the ways of working out problems, will be very useful to you in this study. There are two things the introduction of which really makes all the difference between arithmetic and algebra. One of these is the use of *letters to represent numbers*, and you will see in the following exercises that this change makes the solution of problems much easier.

Exercise I.

Illustrative Example. The sum of two numbers is 60, and the greater is four times the less. What are the numbers?

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Solution.

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Let $x=$ the less number;
then $4x=$ the greater number,
and $4x + x = 60$,
or $5x = 60$;
therefore $x = 12$,
and $4x = 48$.   The numbers are 12 and 48.

1. The greater of two numbers is twice the less, and the sum of the numbers is 129. What are the numbers?

2. A man bought a horse and carriage for $500, paying three times as much for the carriage as for the horse. How much did each cost?

3. Two brothers, counting their money, found that together they had $186, and that John had five times as much as Charles. How much had each?

4. Divide the number 64 into two parts so that one part shall be seven times the other.

5. A man walked 24 miles in a day. If he walked twice as far in the forenoon as in the afternoon, how far did he walk in the afternoon?

6. For 72 cents Martha bought some needles and thread, paying eight times as much for the thread as for the needles. How much did she pay for each?

7. In a school there are 672 pupils. If there are twice as many boys as girls, how many boys are there?

Illustrative Example. If the difference between two numbers is 48, and one number is five times the other, what are the numbers?

Solution.

Let	$x=$ the less number;
then	$5x=$ the greater number,
and	$5x - x = 48,$
or	$4x = 48;$
therefore	$x = 12,$
and	$5x = 60.$

The numbers are 12 and 60.

8. Find two numbers such that their difference is 250 and one is eleven times the other.

9. James gathered 12 quarts of nuts more than Henry gathered. How many did each gather if James gathered three times as many as Henry?

10. A house cost $2880 more than a lot of land, and five times the cost of the lot equals the cost of the house. What was the cost of each?

11. Mr. A. is 48 years older than his son, but he is only three times as old. How old is each?

12. Two farms differ by 250 acres, and one is six times as large as the other. How many acres in each?

13. William paid eight times as much for a dictionary as for a rhetoric. If the difference in price was $6.30, how much did he pay for each?

14. The sum of two numbers is 4256, and one is 37 times as great as the other. What are the numbers?

15. Aleck has 48 cents more than Arthur, and seven times Arthur's money equals Aleck's. How much has each?

16. The sum of the ages of a mother and daughter is 32 years, and the age of the mother is seven times that of the daughter. What is the age of each?

17. John's age is three times that of Mary, and he is 10 years older. What is the age of each?

Exercise 2.

Illustrative Example. There are three numbers whose sum is 96; the second is three times the first, and the third is four times the first. What are the numbers?

<div align="center">

Solution.

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$$\text{Let} \qquad x=\text{first number,}$$
$$3x=\text{second number,}$$
$$4x=\text{third number.}$$
$$x + 3x + 4x=96$$
$$8x=90$$
$$x=12$$
$$3x=36$$
$$4x=48$$

The numbers are 12, 36, and 48.

1. A man bought a hat, a pair of boots, and a necktie for $7.50; the hat cost four times as much as the necktie, and the boots cost five times as much as the necktie. What was the cost of each?

2. A man traveled 90 miles in three days. If he traveled twice as far the first day as he did the third, and three times as far the second day as the third, how far did he go each day?

3. James had 30 marbles. He gave a certain number to his sister, twice as many to his brother, and had three times as many left as he gave his sister. How many did each then have?

4. A farmer bought a horse, cow, and pig for $90. If he paid three times as much for the cow as for the pig, and five times as much for the horse as for the pig, what was the price of each?

5. A had seven times as many apples, and B three times as many as C had. If they all together had 55 apples, how many had each?

6. The difference between two numbers is 36, and one is four times the other. What are the numbers?

7. In a company of 48 people there is one man to each five women. How many are there of each?

8. A man left $1400 to be distributed among three sons in such a way that James was to receive double what John received, and John double what Henry received. How much did each receive?

9. A field containing 45,000 feet was divided into three lots so that the second lot was three times the first, and the third twice the second. How large was each lot?

10. There are 120 pigeons in three flocks. In the second there are three times as many as in the first, and in the third as many as in the first and second combined. How many pigeons in each flock?

11. Divide 209 into three parts so that the first part shall be five times the second, and the second three times the third.

12. Three men, A, B, and C, earned $110; A earned four times as much as B, and C as much as both A and B. How much did each earn?

13. A farmer bought a horse, a cow, and a calf for $72; the cow cost twice as much as the calf, and the horse three times as much as the cow. What was the cost of each?

14. A cistern, containing 1200 gallons of water, is emptied by two pipes in two hours. One pipe discharges three times as many gallons per hour as the other. How many gallons does each pipe discharge in an hour?

15. A butcher bought a cow and a lamb, paying six times as much for the cow as for the lamb, and the difference of the prices was $25. How much did he pay for each?

16. A grocer sold one pound of tea and two pounds of coffee for $1.50, and the price of the tea per pound was three times that of the coffee. What was the price of each?

17. By will Mrs. Cabot was to receive five times as much as her son Henry. If Henry received $20,000 less than his mother, how much did each receive?

Exercise 3.

Illustrative Example. Divide the number 126 into two parts such that one part is 8 more than the other.

<div align="center">

Solution

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$$\text{Let} \qquad x = \text{less part,}$$
$$x + 8 = \text{greater part.}$$
$$x + x + 8 = 126$$
$$2x + 8 = 126$$
$$2x = 118^{1}$$
$$x = 59$$
$$x + 8 = 67$$

The parts are 59 and 67.

1. In a class of 35 pupils there are 7 more girls than boys. How many are there of each?

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[1] Where in arithmetic did you learn the principle applied in transposing the 8?

2. The sum of the ages of two brothers is 43 years, and one of them is 15 years older than the other. Find their ages.

3. At an election in which 1079 votes were cast the successful candidate had a majority of 95. How many votes did each of the two candidates receive?

4. Divide the number 70 into two parts, such that one part shall be 26 less than the other part.

5. John and Henry together have 143 marbles. If I should give Henry 15 more, he would have just as many as John. How many has each?

6. In a storehouse containing 57 barrels there are 3 less barrels of flour than of meal. How many of each?

7. A man whose herd of cows numbered 63 had 17 more Jerseys than Holsteins. How many had he of each?

8. Two men whose wages differ by 8 dollars receive both together $44 per month. How much does each receive?

9. Find two numbers whose sum is 99 and whose difference is 19.

10. The sum of three numbers is 56; the second is 3 more than the first, and the third 5 more than the first. What are the numbers?

11. Divide 62 into three parts such that the first part is 4 more than the second, and the third 7 more than the second.

12. Three men together received $34,200; if the second received $1500 more than the first, and the third $1200 more than the second, how much did each receive?

13. Divide 65 into three parts such that the second part is 17 more than the first part, and the third 15 less than the first.

14. A man had 95 sheep in three flocks. In the first flock there were 23 more than in the second, and in the third flock 12 less than in the second. How many sheep in each flock?

15. In an election, in which 1073 ballots were cast, Mr. A receives 97 votes less than Mr. B, and Mr. C 120 votes more than Mr. B. How many votes did each receive?

16. A man owns three farms. In the first there are 5 acres more than in the second and 7 acres less than in the third. If there are 53 acres in all the farms together, how many acres are there in each farm?

17. Divide 111 into three parts so that the first part shall be 16 more than the second and 19 less than the third.

18. Three firms lost $118,000 by fire. The second firm lost $6000 less than the first and $20,000 more than the third. What was each firm's loss?

24. In how many days can A, B, and C build a boat if they work together, provided A alone can build it in 24 days, B in 18 days, and C in 30 days?

The above method of solving affected quadratic equations is the simplest of three methods commonly used, and will not solve all possible cases; the method given for solving simultaneous equations is only one of three known methods; the cases in factoring are less than half of those usually taken. In fact, we have made only a beginning in the subject of algebra; much more lies ahead along the lines which we have been following. *Can you grasp more clearly the conditions given in any problem presented to you, and see more definitely just what is required, than when you began this study? Do you possess greater ability to think out problems? Has the use of letters to represent numbers made you think more exactly what is to be done, and what the operations mean?* If so, your knowledge of numbers is broader, and you already know that

Algebra is the knowledge which has for its object general truths about numbers.

Exercise 58. (General Review.)

1. When $a = l$, $b = 3$, $c = 5$, and $d = O$, what is the value of
$$\frac{4a + b^2 + b^2c^2 + ad}{b^2 + c^2 + d^2} - \frac{1 + a^2c^2}{a^2 + c^2 + d^2} + \frac{a^2 + b^2 + d^2}{1 + a^2b^2 + bd} - \frac{a^2 + 2ab + b^2}{b^2 - 2bc + c^2}?$$

2. Prove that $(x^2 + xy + y^2)(x^2 - xy + y^2) = \dfrac{x^6 - y^6}{x^2 - y^2}$

3. Solve $(x + 5)^2 - (x + 1)^2 - 16x = (x - 1)^2 - (x - 5)^2$.

4. A tank is filled by two pipes, A and B, running together, in 12 hours, and by the pipe B alone in 20 hours. In what time will the pipe A alone fill it?

5. Find the G.C.F. of $x^3 + 1 - x - x^2$, $x^3 + x - 1 - x^2$, $x^4 - 1$, and $x^2 - 4x + 3$.

6. Divide $a^5 + a^4b - a^3b^2 + a^3 - 2ab^2 + b^3$ by $a^3 - b + a$.

7. Find the square root of $5y^4 + 1 + 12y^5 - 2y - 2y^3 + 4y^6 + 7y^2$.

8. Expand
$$(x + 1)(x + 2) - (2x + 1)(2x + 3) + (x - 4)(x - 9) + (x - 5)^2.$$

9. Solve $\dfrac{x + 2}{x - 2} + \dfrac{x - 2}{x + 2} = \dfrac{5}{2}$.

10. Simplify
$$\left(\frac{1}{x - 1} - \frac{3}{(x + 3)(x - 1)}\right) \div \left(\frac{1}{x + 3} + \frac{1}{(x - 1)(x + 3)}\right).$$

II.

11. Add $2(a-c)^3 - 10x^3y - 7(a-c)$, $6(a-c) - 2(a-c)^3 - 10x^3y$, $3(a-c) - (a-c)^3 + 2x^3y$, $2(a-c) + x^3y - (a-c)^3$, $4(a-c) + 5(a-c)^3 + 2x^3y$, $3(a-c) - 2x^3y - 6(a-c)^3$.

12. Solve $bx - b^2 = 3b^2 - 4bx$.

13. Factor $x^6 + 2x^3 - 3$, $ax^2 - ay^2 + by^2 - bx^2$, $27x^3 + (y+z)^3$.

14. Find the fraction which becomes equal to one when six is added to the numerator, and equal to one-third when four is added to the denominator.

15. Simplify $\dfrac{\frac{a^2}{b^3} + \frac{1}{a}}{\frac{a}{b^3} - \frac{1}{b} + \frac{1}{ab}}$.

16. Solve $\dfrac{7x+9}{4} = \left(x - \dfrac{2x-1}{9}\right) + 7$.

17. Six years ago John was five times as old as Sarah. If he is twice as old as Sarah now, what are their ages?

18. Multiply together $\frac{1-x^2}{1+y}$, $\frac{1-y^2}{x+x^2}$, and $1 + \frac{x}{1-x}$.

19. Simplify $a^2 - (b^2 - c^2) - \{b^2 - (c^2 - a^2)\} + \{c^2 - (b^2 - a^2)\}$.

20. x times y is how many times a?

III.

21. Add $2x + y - 2a + 55\frac{1}{2}b$, $24b - y + 2x + a$, $3a - 2y - 4x - 81b$, and subtract the result from $2y + 3a + \frac{1}{2}b + 3x$.

22. Divide $\frac{11}{8}a^2 - \frac{5}{4}a^3 - \frac{1}{2}a + a^4$ by $a^2 - \frac{1}{2}a$.

23. A can do a piece of work in 3 days which B can do in 5 days. In what time can they do it working together?

24. Simplify $\dfrac{a-b}{a^2 - ab + b^2} + \dfrac{ab}{a^3 + b^3} + \dfrac{1}{a+b}$.

25. Factor $x^2 - 9x - 52$, $1 - a^{16}$, $(a^2 + b^2)^2 + 2(a^4 - b^4) + (a^2 - b^2)^2$.

26. Solve $\dfrac{3x}{4} - \dfrac{x-10}{2} = x - 6 - \dfrac{x-4}{2}$.

27. The sum of the ages of a man and his son is 100 years; one-tenth of the product of their ages exceeds the father's age by 180. How old are they?

28. Solve $x = 9 - \frac{y}{2}$, $y = 11 + \frac{x}{3}$.

29. From what must $3x^4 - 2x^3 + x - 6$ be subtracted to produce unity?

30. Find the following roots: $\sqrt{5.5225}$, $\sqrt[3]{32.768}$.

IV.

31. Find the value of $\dfrac{4x^3 + 2y^3}{ab} + \dfrac{2y^3 + 4z^3}{z^3 + y^2} - \dfrac{b^3 - z^2 b}{a^2 b}$, if $x = 1$, $y = 2$, $z = 0$, $a = 4$, and $b = 5$.

32. Solve $\dfrac{4}{x - 6} - \dfrac{3}{x - 9} = \dfrac{1}{x - 3}$.

33. Find three consecutive numbers whose sum is 78.

34. Find the G.C.F. of $2a^3 - 12a - 2a^2$, $a^4 - 4a^2$ and $4a^3 b + 16ab + 16a^2 b$.

35. Divide $\dfrac{x^4 - y^4}{x^2 - 2xy + y^2}$ by $\dfrac{x^2 + xy}{x - y}$.

36. A fraction becomes $\frac{3}{4}$ by the addition of three to the numerator and one to the denominator. If one is subtracted from the numerator and three from the denominator, it becomes $\frac{1}{2}$. What is the fraction?

37. Expand $\left(\dfrac{3a^2 b (m+n)^2}{4xy^3} (a - b)^3 \right)^3$, $\sqrt{\dfrac{50x^4 (a+b)^7}{32y^6 z^2 (a+b)}}$.

38. If a certain number is multiplied by itself, the result is $9x^4 - 4x + 10x^2 + 1 - 12x^3$. Find the number.

39. Simplify $\dfrac{ax - x^2}{(a + x)^2} \times \dfrac{a^2 + ax}{(a - x)^2} \div \dfrac{2ax}{a^2 - x^2}$.

40. Solve $18x - 20y = 3$, $\dfrac{4y - 2}{3} - \dfrac{5x}{2} = 0$.

V.

41. Factor $x^4 + 5x^2 + 6$, $x^2 - 14x + 49$, $x^2 - (y + z)^2$.

42. Add $xy - \frac{9}{8}x - \frac{7}{12}(x^2 - y^2) - 5x^2 y^2$, $\frac{5}{8}x - xy + 9x^2 y^2 + \frac{2}{3}(x^2 - y^2)$, $\frac{1}{9}x^2 y^2 - xy + \frac{1}{4}x + \frac{3}{4}(x^2 - y^2)$, $2xy + \frac{1}{4}x - \frac{5}{6}(x^2 - y^2) - 4x^2 y^2$.

43. At what times between 7 and 8 o'clock are the hands of a clock six minutes apart?

44. Simplify $\dfrac{x^2 - 5x + 6}{x^2 - 2x + 1} \times \dfrac{x^2 - 4x + 3}{x^2 - 4x + 4} \div \dfrac{x^2 - 6x + 9}{x^2 - 3x + 2}$.

45. Solve $\dfrac{x + 2}{b + 2} = 2 - \dfrac{x + 1}{b + 1}$.

46. Factor $\dfrac{x^6}{y^6} - \dfrac{a^2 b^4}{c^2}$, $\dfrac{x^2}{y^2} - \dfrac{5x}{y} - 14$, $\dfrac{x^2}{y^2} - 2 + \dfrac{y^2}{x^2}$.

47. A, who works only two-thirds as fast as B, can build a stone wall in 12 days. In what time could A and B together build the wall?

48. Solve $\dfrac{x + y}{2} - \dfrac{x - y}{3} = 8$, $\frac{x+y}{3} + \frac{x-y}{4} = 11$.

49. Expand $(1 + 2x)^3$, $\left(2x^2 - 3a^2b^3\right)^4$.

50. Reduce $\dfrac{(a^4 + 2a^2b^2 + b^4)(a^4 + b^4)}{a^8 - b^8}$ to lowest terms.

VI.

51. y is how much greater than x?

52. Subtract $3x^3 + 4x^2y - 7xy^2 + 10y^3$ from $4x^3 - 2x^2y + 4xy^2 + 4y^3$ and find the value of the remainder when $x = 2$ and $y = 1$.

53. The length and width of a rectangle are respectively 5 feet longer and 4 feet shorter than the side of an equivalent square. What is its area?

54. Find the L.C.M. of $a^2 - 3 - 2a$, $a^2 - 1$, and $2a^2 - 6a + 4$.

55. Simplify $\dfrac{\frac{b}{4a} - 1 + \frac{a}{b}}{\frac{b}{2a} - \frac{2a}{b}}$.

56. Solve $\dfrac{x - 1}{3} + \dfrac{3}{x - 1} = 2$.

57. Factor $a^4b + 8ac^3bm^6$, $4c^3x^2 + cy^2 + 4c^2xy$, $x^6 - 1$.

58. Multiply $1 - \dfrac{1}{2}x - \dfrac{1}{3}x^2 + \dfrac{1}{4}x^3$ by $1 - \frac{1}{3}x^2 - \frac{1}{4}x^3 - \frac{1}{2}x$.

59. Find the cube root of $6x^4 + 7x^3 + 3x^5 + 6x^2 + x^6 + 1 + 3x$.

60. Divide $12x^2y^2 - 4y^4 - 6x^3y + x^4$ by $x^2 + 2y^2 - 3xy$.

VII.

61. Add $\frac{1}{10}a^3 - \frac{4}{5}a^4 - \frac{1}{5}a^2 + \frac{3}{10}a$, $\frac{1}{4}a^2 - \frac{4}{5}a - \frac{5}{7}a^4 - \frac{1}{8}a^3$, $\frac{5}{7}a^4 + \frac{1}{8}a^3 + \frac{3}{4}a^2 + \frac{2}{5}a$, $\frac{4}{5}a^4 + \frac{1}{5}a^2 + \frac{2}{5}a^3 + \frac{1}{10}a$.

62. Solve $x(a - x) + x(b - x) = 2(x - a)(b - x)$.

63. Factor $x^4 - 22x^2 - 75$, $16 - x^8$, $(a + b)^2 - (a - b)^2$.

64. A piece of work can be finished by 3 men in 8 days, or by 5 women in 6 days, or by 6 boys in 6 days. In what time can 2 men, 3 women, and 3 boys do the work?

65. Solve $\dfrac{3x + 19}{2} - \left(\dfrac{x + 1}{6} + 3\right) = \dfrac{5x + 2}{3} - \left(3 - \dfrac{3x - 1}{2}\right)$.

66. Expand $\left(\dfrac{a}{b} - \dfrac{c}{d}\right)^3$, $\left(\dfrac{c}{d} + 1\right)\left(\dfrac{c^2}{d^2} - \dfrac{c}{d} + 1\right)$.

67. What number is that, the sum of whose third and fourth parts is less by two than the square of its sixth part?

68. Solve $\dfrac{x}{5} - \dfrac{y}{7} = 1$, $\frac{2x}{3} - \frac{y}{2} = 3$.

69. Divide m by $1 + y$ to four terms.

70. If x is $\frac{3}{5}$ of a number, what is the number?

VIII.

71. The head of a fish is 6 inches long, the tail is as long as the head and half the body, and the body is as long as the head and tail. What is the length of the fish?

72. Add $4a - 5x - 15y$, $a + 18x + 8y$, $4a - 7x + 11y$, $a + 3x + 5y$, and multiply the result by the difference between $11a + 7y$ and $10a + 6y - x$.

73. Divide $2x^2 + \frac{9}{2}x^4 + \frac{8}{9}$ by $2x + 3x^2 + \frac{4}{3}$.

74. How many numbers each equal to $1 - 2x + x^2$ must be added together to equal $5x^6 - 6x^5 + 1$?

75. Factor $a^3 + 5a^2 - 4a - 20$, $x^6 - y^6$, $2x^5 - 8x^3y^2 + 6xy^4$.

76. A courier who travels at the rate of 5 miles an hour is followed, 4 hours later, by another who travels at the rate of 15 miles in 2 hours. In how many hours will the second overtake the first?

77. Divide $\dfrac{1}{1-x} - \dfrac{1}{1+x}$ by $\dfrac{1}{1-x} + \dfrac{1}{1+x}$.

78. Solve $3x - 4y = -6$, $10x + 2y = 26$.

79. $3xy - 3a^2 + 4b^2 - 5cd + 4xy - 6a^2 - 7b^2 + 7cd + 3xy - 6a^2 + 6b^2 - 3cd - 5xy + 7a^2 - 6b^2 + 4cd + 4xy + 7a^2 - 7b^2 + 4cd - 6xy - 6a^2 + 3b^2 - 7cd + 7a^2 = ?$

80. Simplify $3x - 5 - \{2(4 - x) - 3(x - 2)\} + \{3 - (5 + 2x) - 2\}$.

ANSWERS TO A FIRST BOOK IN ALGEBRA.

Exercise 1.

1. 43; 86.

2. Carriage, $375; horse, $125.

3. C, $31; J, $155.

4. 8; 56.

5. 8 miles.

6. Needles, 8; thread, 64.

7. 224 girls; 448 boys.

8. 25; 275.

9. H, 6 qts.; J, 18 qts.

10. Lot, $720; house, $3600.

11. Mr. A, 72; son, 24.

12. 50 A.; 300 A.

13. Dict., $7.20; rhet, $.90.

14. 112; 4144.

15. Aleck, 56; Arthur, 8.

16. Mother, 28; daughter, 4.

17. J, 15 yrs.; M, 5 yrs.